

$$f: x \rightarrow \frac{1}{x(1-\ln x)}$$

1a $\mathbb{D}_f: x > 0; x \neq 0$ $\mathbb{D}_g: \mathbb{R}^+ \setminus \{e\}$
 $\ln x \neq 1 \Rightarrow e \neq x$

$$\lim_{x \rightarrow 0^+} \frac{1}{x(1-\ln x)} = \infty; \lim_{x \rightarrow \infty} \frac{1}{x(1-\ln x)} = 0^-$$

$$\lim_{x \rightarrow e^-} \frac{1}{x(1-\ln x)} = +\infty$$

1b $f'(x) = \frac{0 \cdot x(1-\ln x) - 1 \cdot [x(1-\ln x)]'}{[x(1-\ln x)]^2}$ $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$
 $= \frac{-1 \cdot \ln x}{[x(1-\ln x)]^2} = [f(x)]^2 \cdot \ln x$ $(uv)' = u'v + v'u$

$$f'(x) = \frac{[f(x)]^2 \cdot \ln x}{>0}; f' > 0 \text{ für } \ln x > 0 \Rightarrow x > 1 \text{ str. mono. wa}$$

$$f' < 0 \text{ für } \ln x < 0 \Rightarrow 0 < x < 1 \text{ str. mono. fall}$$

\Rightarrow Min bei $x^E = 1; y^E = 1$

$$h(x) = \frac{2}{x} \quad h'(x) = -\frac{2}{x^2}$$

2a Schnittpkt. zweier Funktionen $g, h: g=h$

⑧ $\frac{1}{x(1-\ln x)} = \frac{2}{x} \Rightarrow x = 2x(1-\ln x)$

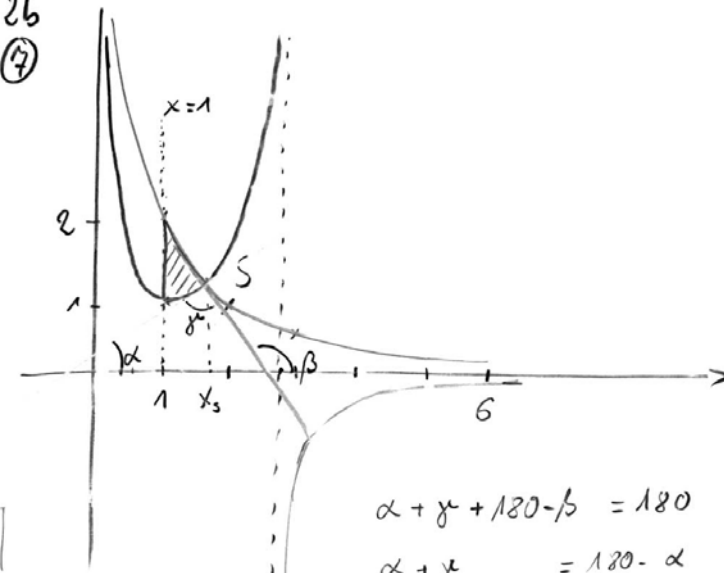
$$x = 2x - 2x \ln x \Rightarrow 2x \ln x = x \quad | : x \neq 0$$

$$2 \ln x = \frac{1}{2} \Rightarrow \boxed{x_S = e^{\frac{1}{2}}} \quad \boxed{y_S = \frac{2}{e^{\frac{1}{2}}} = 2e^{-\frac{1}{2}}}$$

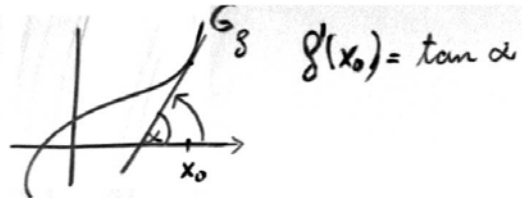
$$f'(e^{\frac{1}{2}}) = \tan \alpha = \left[\frac{1}{e^{\frac{1}{2}} \cdot \frac{1}{2}} \right]^2 \cdot \frac{1}{2} = \frac{2}{e}; \quad h'(e^{\frac{1}{2}}) = \tan \beta = -\frac{2}{e}$$

$\alpha = 36,3^\circ$ $\beta = 180^\circ - \alpha$

2b
⑦



$$\begin{aligned} \alpha + \beta + 180 - \beta &= 180 \\ \alpha + \beta &= 180 - \alpha \\ \beta &= 180 - 2\alpha \\ \beta &= 107,4^\circ \end{aligned}$$

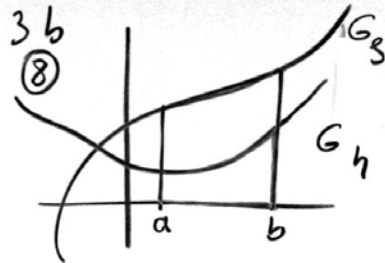


3a $F: x \rightarrow -\ln(1-\ln x)$

③ Eine Fkt $F(x)$ ist Stammfkt. zu einer Funktion $f(x)$ wenn gilt: $F'(x) = f(x)$

$$\ln g(x)' = \frac{1}{g(x)} \cdot g'(x)$$

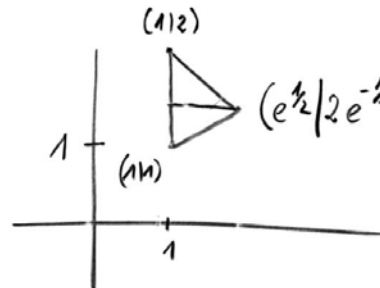
$$F' = -\frac{1}{1-\ln x} \cdot \left(-\frac{1}{x}\right) = \frac{1}{x(1-\ln x)} = f(x)$$



Fläche zw. den beiden Graphen G_f u G_h zw. a u $b = \int_a^b (f-h) dx$

$$\int_1^{e^{\frac{1}{2}}} \left(\frac{2}{x} - \frac{1}{x(1-\ln x)} \right) dx = \left[2 \ln x + \ln(1-\ln x) \right]_1^{e^{\frac{1}{2}}}$$

$$= \left[1 + \ln \frac{1}{2} \right] - 0$$



$$F_A = \frac{1}{2} \cdot 1 \cdot (e^{\frac{1}{2}} - 1)$$

$$\frac{\frac{1}{2}(e^{\frac{1}{2}} - 1) - (1 + \ln \frac{1}{2})}{1 + \ln \frac{1}{2}} \cdot 100 \% = 5,7\%$$